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Book Reviews

Cosserat Theories: Shells, Rods and Points

M. B. Rubin, Kluwer Academic Publishers, 2000, Dordrecht, The Netherlands, 496 pp., $205.00

The purpose of structural mechanics is to characterize the mechanical response of different elements used in structural applications by reducing the general three-dimensional theory of continuum mechanics that models a solid's behavior. Examples of structural elements of interest are beams, rods, plates, and shells. The resulting reduced theories take into account specific characteristics of the geometry and loading of these elements. In this way, a shell is characterized by one direction, the transverse or thickness direction, the scale of which is much smaller than that of the other two spanning the shell's midsurface; a rod consists of an axis and a cross section with characteristic dimensions smaller than the length of the axis itself. The three-dimensional displacement field that determines the general response of a classical continuum is then reduced to a set of generalized displacements characterizing the motion of these different objects. The problem is thereby reduced in dimensions from a partial differential equation in the three-dimensional continuum to even an ordinary differential equation in space, as in the case of a beam or rod. This reduction not only permits solution of the mathematical problem in many particular situations of practical interest but also leads to a better understanding of the response of the physical system. In this way, the notions of axial forces, bending moments, twisting torques, etc., are as important for structural engineers as the fundamental notion of stress.

A main challenge in this reduction process appears when one is interested in the consideration of finite deformations and strains in the solid. This arises especially because of the need of the final theory to be frame indifferent. This physical requirement states that the deformation of the solid should be independent of the observer, a concept understood in the classical sense of a superposed rigid body motion. Classical infinitesimal theories, and even some nonlinear theories, are not frame indifferent under superposed finite rigid body motions and therefore are limited in the range of applications involving finite deformations. The Cosserat theories of structural mechanics (referred to in this way by virtue of the contributions of the Cosserat brothers in the early years of the 20th century) are able to model the structural systems of interest and incorporate this fundamental physical property.

The basic characteristic of a Cosserat theory is the consideration of the so-called directors. These are vectors that define the response of the solid in the aforementioned characteristic directions identified in its structural definition. In this way, a Cosserat shell is characterized in its lower form by a single director field defined over the shell's midsurface. Similarly, a Cosserat rod consists of two director fields along the rod's axis defining the motion of the cross section. A lower theory refers to the fact that many additional levels of reduction can be considered from the general three-dimensional continuum, incorporating in this way as many strain components from the general response of the solid as desired. In fact, we can also find a general three-dimensional Cosserat continuum incorporating, in a pointwise fashion, the deformation of a vector frame to characterize, for example, properties of the microstructure of the material. The current book does not go into this particular case and focuses on the structural theories of Cosserat shells and rods. It also considers the abstraction of a Cosserat point, as a solid of three small dimensions and characterized uniquely by the deformation of a director frame. As indicated later, the author uses this abstraction in the development of interesting numerical methods for the solution of general problems in solid mechanics.

Different constraints on these directors lead to different useful theories. For example, the consideration of two orthonormal directors during the whole deformation process of the rod leads to the usual assumption of the cross section remaining plane without distortion. The incorporation of the well-known Kirchhoff assumption of sections remaining orthogonal to the deformed axis can also be accomplished easily in this context. All together, and despite the aforementioned reduction in the dimensionality of the problem, the complexity of the problem increases notably due to the need to treat these geometrical entities, the directors with all of these possible constraints, correctly so the basic invariance property is preserved. Despite this complexity, there exists nowadays a strong interest in the formulation and analysis of these nonlinear structural theories. They are referred to by many authors as "geometrically exact theories" because no limit exists on the range of the different displacement variables in contrast to infinitesimal theories. One of the main reasons for consideration of these theories in the past 20 years has been the desire for numerical solutions of the resulting initial boundary value problems. The finite element method has been a common choice for this purpose. The present book does not cover this particular aspect, although the author cites fundamental references of these works to achieve a comprehensive manuscript.

The main focus of the book is on the development of the theories themselves. In particular, the author accomplishes his goal of presenting the Cosserat theories with the minimum of mathematical constructions. Even though a quick look at the book may be overwhelming in the number and length of the mathematical formulas, a careful reading shows the ease of following different mathematical developments. This should be contrasted with classical references in this area that make extensive use of more involved mathematical constructions. (For example, the use of Christoffel symbols is avoided completely in the present book.) It conforms in this way with the modern treatment of these theories in the applied and computational mechanics literature. We should note, however, that the book is directed to researchers already competent in nonlinear continuum mechanics. Even though the book includes a chapter in this basic area, many details (especially proofs of fundamental results) are left out, under the assumption that they are known to the reader.

A distinctive characteristic of the book compared with existing references in the field is the full characterization of the constitutive relations in the general Cosserat theories under consideration. In this way, the book identifies not only the strain measures characterizing the deformation of the particular structural member at hand but also the appropriate relation between these strain measures and the associated stress resultants (e.g., curvature and bending moment) in the general case of nonlinear othotropic materials. This is not an easy task in a general finite deformation range, but the author accomplishes it in an excellent and rigorous manner. The hyperelastic range is assumed in the developments and is determined by a stored energy function defining an elastic potential. Uncoupled thermomechanical conditions are considered throughout the book. An important outcome of this approach is the possibility of direct application of the mathematical theories developed in the book to practical applications.

A more detailed outline of the book is as follows. Chapter 1 presents a brief introduction to the material presented in the rest of the book. In particular, the structure of the book is outlined in detail. We also find a long list of references in this first chapter, covering even material not discussed in the book but where Cosserat theories have been employed successfully beyond structural mechanics, as indicated earlier. The later developments in the book do not make much mention of other references in the field, and so the reader may need to refer to this first chapter and its discussion of the literature to capture a full picture of the field. Chapter 2 presents a very brief summary of some mathematical fundamentals. The author focuses his attention on basic results of tensor algebra and analysis in general curvilinear coordinates. This brief chapter is supplemented by the two appendices at the end of the book.

Chapter 3 concludes the introductory part of the book by presenting a summary of nonlinear continuum mechanics in the general three-dimensional case. The author presents several specific problems with a closed-form solution in the linearized range (e.g., pure bending and torsion of a rectangular parallelepiped, plane strain vibrations of an isotropic circular cylinder, etc.) to use later in the calibration of the structural models. As indicated earlier, the presentation is kept to a minimum, assuming that the reader is familiar with most of the described material. In fact, the author includes several exercises at the end of the book referring to the different sections in each chapter, asking mostly for derivations of the different formulas and further elaboration of the results presented. This approach is in accordance with his intentions of keeping the mathematical developments to a minimum.

The following three chapters (4, 5, and 6) describe the theories of Cosserat shells, rods, and points, respectively. These three chapters are completely independent of each other, with the reader being able to move to any of them as desired. In fact, the three chapters follow the same outline in exposition, including the notation employed. This is an especially well-thought-out characteristic of the book, because it permits understanding of developments in one case to be used in understanding other cases. Each of these chapters starts with a description of the geometric characterization of the structural member under consideration through the proper choice of the director fields. Then the general mechanical theory is developed following several approaches, from a direct postulation of the governing equations to their derivation from the equations of nonlinear continuum mechanics presented in Chapter 3. The frame indifference of the theories under superposed rigid body motions is discussed in detail. Next the author develops the constitutive relations by matching the response obtained in homogeneous deformations and the benchmark problems solved previously in closed form in the linearized three-dimensional problem. After a brief section summarizing the general equations considered so far, these three chapters close by considering special cases that different constraints in the assumed director fields lead to. In this way, we can see the analysis of membranes at the end of Chapter 4 and the analysis of Bernoulli–Euler rods and Timoshenko rod and strings at the end of Chapter 5. The independence and parallelism of these three chapters is again noted here.

The book concludes with Chapter 7, where the author presents several numerical techniques based on the concept underlying the previously developed Cosserat theories. For example, the solution of spherically symmetric continuum problems is obtained through the discretization of the continuum in a series of contiguous spherical Cosserat shells; the motion of a string, a rod, and even of a three- or two-dimensional continuum is obtained numerically by discretizing the system in smaller pieces having the characteristics of a Cosserat point (that is, with a structure based on a set of directors). Although not investigated in the book, the links with existing numerical techniques seem clear, such as the understanding of a typical tetrahedron employed in the finite element discretization of a continuum as defining a constant state of strain through the deformations of its edges (the directors in the author's approach). Even though some numerical results are presented in the numerical treatment of spherically symmetric continuum problems, rods and strings, these methods seem to be under further development by the author, requiring a still more detailed analysis of its numerical performance. The reader should also refer to the paper cited by the author in the beginning of this chapter describing more standard finite element techniques for the solutions of the problem at hand.

In conclusion, this is a well-written book that covers important aspects of an area of intensive study in current continuum and structural mechanics research. It is a good example of a modern theoretical treatment of the Cosserat theories of structural mechanics and differs from the few existing books in the field by its focus on the mechanical description of the systems of interest through a more simplified mathematical presentation. The researchers in the field will find an excellent collection of important material in the formulation of these nonlinear theories.

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Shape and Structure, from Engineering to Nature

A. Bejan, Cambridge University Press, New York, 2000, 324 pp., $39.95

This is a most unusual, and fascinating, book. Adrian Bejan is a heat transfer engineer at Duke University and the author of 10 books on heat transfer and thermodynamics. In this book he takes a very broad look at optimization. In nature we might call it evolution—one could easily mistake a shark for a dolphin (probably only once…) although they are vastly different species; and an oak tree looks remarkably like a lung. There is not much latitude in the design of a streamlined body, and the tree and the lung are both solutions, broadly speaking, of the problem of providing the maximum area within the minimum volume. Aristotle argued that the characteristics and behavior of plants (particularly) and animals were the result of their "nature," implying a deterministic process of optimization rather than the near-random process of evolution. If he were right, the similarity between engineering and nature would be even closer than Dr. Bejan makes it out to be.

Dr. Bejan says at the outset that he does not intend to discuss the work of others. This is a pity in at least one respect because it means that there are merely a few citations, and no discussion, of D'Arcy Thompson's classic book *Growth and Form*, last reprinted, in the full 1116-page edition and an abridgment of roughly half the size, by Cambridge University Press in 1992. *Growth and Form* is clearly an ancestor of the present book. Thompson was a biologist, but his outlook was much the same as Bejan's, and he did not neglect similarities between creatures and artifacts—his book contains a remarkable photograph of the main wing bone of a vulture that is, quite simply, a Warren truss.

The book is so wide ranging that a list of chapter contents would be somewhat incomprehensible. Many of Dr. Bejan's examples are taken from heat transfer, particularly the design of optimum heat exchangers, and from fluid mechanics. Several chapters are based on the general concept of a "tree" (a real tree, not a mathematical one). One example is the problem of cooling a more-or-less uniformly heated volume (e.g., a microchip) and finally extracting the heat through one conductor or fluid-flow duct. Dr. Bejan calls this the "volume-to-point flow problem," and it is closely analogous to the problem of designing a real tree, or a lung. There are many illustrated discussions of natural fluid flows and multiphase systems. I found the chapter on "Turbulent Structure" rather disappointing: evidently I was expecting too much. Most of the discussion in that chapter relates to cellular convection. More interesting is the discussion of river drainage basins—two-dimensional trees, of course.

The book is not intended as a formal treatise on optimization theory, and one cannot imagine it being used as the main textbook of a course, but it is very clear in its identification of objectives and of constraints, distinguishing global constraints, such as the required lift of a wing, and local constraints, such as the need to avoid excessively thin sections at any part of the span. The mathematics should be within the grasp of a good graduate student in engineering or physics. Incidentally, fractal geometry is quite rightly dismissed on p. 3 as being descriptive, not predictive! Anyone concerned with optimization, including students taking a course on the subject, will derive inspiration and insight from this book, and anyone with an interest in engineering and nature will get a great deal of pleasure from it.

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